Life of Fred®
Complex Analysis

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Notes Before We Begin

Take a peek at the college catalogs of most major universities. Math majors are required to take two years of calculus. Sometime in the third and fourth years every math major must take a course in real analysis and a course in complex analysis.

Let's look at these three subjects.

Calculus introduced the new concept of limits. Virtually everything flowed from $\lim_{x \to a} f(x)$. The variable x stood for some real number.

Real Analysis gave the theoretical underpinnings of calculus. We stayed with the real numbers.

Complex Analysis plays with functions of complex numbers. In particular, we will expand calculus to include complex numbers such as $6 + 7i$. We will do some differentiating and integrating. You may see stuff like $\int f(z)\, dz$.

In the old days you saw $\int f(x)\, dx$ where x was in the real numbers. In this course z will be in the complex numbers.

❁ ❁ ❁

When I am writing, I will use Times New Roman.
When you, my reader, are speaking, ***you will use this font.***
When Fred is talking, he will use this font.

❁ ❁ ❁

Prerequisites? Since we are going to be taking derivatives and antiderivatives, it would be nice if you have had the lower division two years of calculus.

Real analysis, linear algebra, logic, and statistics aren't required, but the more theoretical math you have had, the easier complex analysis will be.

❁ ❁ ❁

The more you enjoy singing, dancing, laughing, and playing, the easier this book will be. We will get complex analysis done, but we will have as much fun as possible. If you are a grouch who just wants to "get it all done," then there are tons of complex analysis books that just give you definition, theorem, proof, definition, theorem, proof, definition, theorem, proof, definition, theorem, proof, definition, theorem, proof, definition, theorem, proof, definition, theorem, proof, definition, theorem, proof, definition, theorem, proof, definition, theorem, proof, definition, theorem, proof till your brain dries up and blows away.

On the other hand not being a quitter will make complex analysis possible for you. Except for Fred, few find this material easy.

In mathematics the stuff you already know seems ten times easier than the stuff you are learning. Do you remember how hard learning to add fractions was? It took weeks of sweat. You barely knew your times tables, and the teacher was yakking* about finding the least common denominator. How were you supposed to find the least common denominator of $\frac{1}{6}$ and $\frac{3}{8}$?

An Aside

Fred didn't have difficulty with this. Working with 6 and 8, he factored both numbers: $2^1 3^1$ and 2^3 and then chose the highest exponent on each prime.

LCD of 6 and 8 is $2^3 3^1 = 24$

For 20, 24, 25, and 27, he factored:

$2^2 5^1$, $2^3 3^1$, 5^2, and 3^3

The LCD is $2^3 3^3 5^2 = 5400$

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* The present progressive form of *yak* can be spelled either *yakking* or *yacking*. English is harder than complex analysis.

Contents

Chapter One
The Conference

Fred's heart felt sore. That's how he expressed it. As the news of his famous teaching technique had spread around the world, people emailed him. They wrote to him. They phoned him. Some went to Kansas to see him in person.

The big question was Complex Analysis. Many were asking when Fred was going to teach it at KITTENS University. They said that if Fred taught it, they would leave their current college and matriculate to KITTENS so they could take it from the master teacher.

Fred looked at his current teaching schedule.

8–9 Arithmetic
9–10 Beginning Algebra
10–11 Advanced Algebra
11–noon Geometry
noon–1 Trigonometry
1–2 Calculus
2–3 Statistics
3–3:05 Break
3:05–4 Linear Algebra
4–5 Seminar in Biology, Economics, Physics, and Metamathematics.

Then he remembered that he was teaching four classes in the early morning.

4–5 Set Theory
5–6 Modern Algebra
6–7 Abstract Arithmetic
7–8 Topology

} These four were added in *Life of Fred: Five Days of Upper Division Math*

The president of KITTENS closed all the buildings at 5 p.m., so Fred couldn't add Complex Analysis to the end of his teaching day. The president also didn't allow classes on the weekend, because it made KITTENS look like it was too dedicated to education. Instead, he rented out the university buildings and grounds to events and conferences. Those things were news. They made the president famous and brought in more money.

So Fred's heart hurt.

The solution was obvious, but it took Fred 0.0003 seconds to figure it out.

It was announced in the school newspaper.

THE KITTEN Caboodle

The Official Campus Newspaper of KITTENS University Thursday 8 a.m. Edition 10¢

exclusive interview

Complex Analysis Conference!

KANSAS: Fred Gauss has just announced you-know-what.

file photo

Caboodle question: When did you think of this?

Fred: 0.0003 seconds ago.

Caboodle question: When and where will it be held?

Fred: On the first Saturday in May, which is two days from now. It will be held in room 123 of the Archimedes Building, the largest auditorium on campus. It starts at 7 a.m.

Caboodle question: Who in the world would want to come to a complex analysis conference on a Saturday at 7 o'clock in the morning?

Fred: Here is a photograph of those who have already signed up.

I will cover a standard course in complex analysis at the conference.

There will be many more attending once the word really gets out.

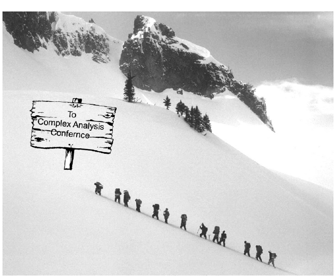
Stop! Hold it. I, your reader, have got a lot of questions before this thing gets out of hand.

This is one of the few math books where the reader can talk back to the author. What's your first question?

Is it true that this Fred is only six years old; that he is that famous; that people are that eager to experience his teaching?

Those are easy questions to answer. Yes, yes, and yes.

People are coming from everywhere.



The newspaper article reported that Fred said that he would "cover a standard course in complex analysis at the conference." We know that newspapers often get things wrong. How can Fred do a complex analysis course in the space of a single day? A standard complex analysis course at the upper division level of a university takes a semester.

Did anyone ever mention to you that Fred is really good at teaching? And, if Fred is up to his usual tricks, he will toss in stories and other poetry/history/music/geography/and so on. As they say, Fred's teaching is like Lake Victoria.*

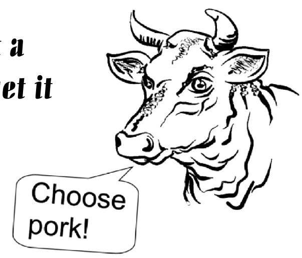
Huh?

It ain't dry.

I just read your footnote. What's the largest freshwater lake in the world?

Lake Superior in North America.

Wait a minute! You distracted me. I've got a real beef** about complex analysis, and I need to get it said before Fred starts his conference on Saturday.



I'm listening.

I've already seen complex numbers. In advanced algebra we solved quadratic equations like $ax^2 + bx + c = 0$ using the quadratic formula. When we got a negative number under the square root sign, we were told that $\sqrt{-1} = i$, and that i is imaginary. I wanted to throw up. Everyone wondered, "What good is that?"

Then in trig they switched from rectangular $a + bi$ to polar form, which was $r(\cos\theta + i \sin\theta)$. I remember changing $\sqrt{3} + i$ into $2 \cos 30^\circ + 2i \sin 30^\circ$, which was called $2 \text{ cis } 30^\circ$.

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* Lake Victoria is the second largest freshwater lake in the world. It is in Africa. Its length from north to south is 210 miles. No one has ever swum that length.

Some people have swum the English channel (21 miles). It was first done in 1875 and took almost 22 hours. In the 1950s people were doing it in about 12 hours. In 2019 (that's almost news) Sarah Thomas took 54 hours and 10 minutes to swim it four times without stopping.

** *Beef* is slang for a complaint.

Then we got hit with de Moivre's theorem so we could do powers and square roots of complex numbers.

And all I could think of was that this stuff was imaginary. It was pixy dust. As useful as learning Icelandic. (Actually, I said stuff that we can't print in this G-rated book.)

I get the feeling that I know where you are heading.

Wrong, Mr. Author. You think I'm going to ask that Fred stick in a bunch of useful stuff in his complex analysis conference. At least some things that will tickle my imagination. De Moivre's raising 5 + 6i to the hundredth power—like we did in trig—just won't do it.

How about . . .

✯ Finding $\log_6(-6)$?

> *An Aside*
>
> You don't have to read any of these "Asides" in this book. I'm writing them for my entertainment **or** to include review material **or** to solve messy equations that no human should have to read.
>
> $\log_2 8$ asks, "What power do you raise 2 to in order to get 8?" $\log_2 8 = 3$.
>
> $\log_{100} 10 = \frac{1}{2}$ since $100^{1/2} = \sqrt{100} = 10$
>
> There is no way that $\log_6(-6)$ makes any sense. Raising 6 to any power will always give you a positive answer.

✯ Raising i to the ith power, i^i, and getting a real number answer!

✯ Finding a number z such that $\sin(z) = 40$.

No you can't. The value of the sine function is always between –1 and 1. My trig book told me that.

And your elementary school books told you that 4 – 7 can't be done.

Lies! Lies! All I get is poppycock.

Or it may be that you weren't ready for the bigger truth.

Take, for example, the Easter Bunny. They may have told you this untruth because you were too young to hear the whole story of some innocent man who was tortured to death.

When you got old enough you may have realized . . .

1. Rabbits don't lay eggs.
2. Laying eggs indicates sexual maturity which bunnies don't have.
3. The brown things that rabbits poop are not chocolate.
4. Those 1-inch diameter chocolate eggs are way too large. They would kill any rabbit who tried to deliver one of them. They would be like a woman trying to give birth to a 40-pound baby.

Again, you got off track—Sarah Thomas and the Easter Bunny! Let me try again. When Fred does his Saturday conference, I don't want him to even mention the imaginary number i. If he wants to talk about the real numbers, that's fine with me. That's calculus-type material. I can handle that.

I understand that Complex Analysis means calculus with the complex numbers like 3 + 8i. (⇦ nice definition)

I have been lied to enough. They <u>defined</u> i by i = $\sqrt{-1}$. This is nuts. I was explicitly told in algebra that $\sqrt{x}$ meant the non-negative number whose square is x.

Is i positive or zero?

No.

Then they gave me the lie that i was defined by $i^2 = -1$. This is also a lousy definition. Except for zero, isn't it always true that x^2 = anything always has two possible answers? Aren't there two different numbers that satisfy $x^2 = -1$?

Yes and yes. When you were told that i was defined by $i^2 = -1$, you were being Easter Bunnied.

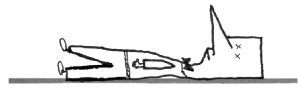
My Demands

1. ***Only real numbers.***
2. ***No phony definitions of i.***

Pass that along to Fred and see what he says.

I will, but I can't promise anything.

When I saw Fred that evening, Fred's response to the demands was what might have been expected.



He passed out

❁ ❁ ❁

As Fred began to wake up, he was mumbling . . .

✡ teach ice skating to people who live in the desert

✡ teach complex analysis using only real numbers 

✡ teach surfing to people in jail

✡ teach complex analysis using only real numbers 

✡ teach logarithms to three-year-olds

✡ teach complex analysis using only real numbers 

Fred awoke and smiled. "No sweat. I was deep into getting ready for the Complex Analysis Conference.* This will take a "slight" adjustment in order to use only real numbers. And no Easter Bunny definitions of i. I'll give them the real thing."

I, your reader, am amazed. I thought I was asking the impossible. Where do I sign up for the conference?

I have already signed you up.

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* Fred was reviewing holomorphic functions. "Holomorphic functions" sounds intimidating *right now*. That's because you haven't attended the conference yet.

Do you remember looking at a calculus book before you took the course? $\int_{x=a}^{b} x^3\,dx$ and $\ddot{y} = 8t$ were utterly mystifying.

Index

Index